Math 142   **Test #1**   **Fall 2011**

You must show all your work to receive full credit, especially when differentiating and integrating functions. If you are using a TI-89, you may use it to check answers, but you must show how you found derivatives and integrals.

1. (35 pts) Find the derivative of each of the following functions.

   (a) $f(x) = e^{3x-1}$

   (b) $f(x) = \ln\left(\dfrac{2x+1}{x+1}\right)$

   (c) $f(x) = \tan 2x$

   (d) $f(x) = \arctan 2x$

   (e) $f(x) = 10^{x^2+x}$

2. (40 pts) Find each of the following definite or indefinite integrals.

   (a) $\displaystyle\int_4^6 \dfrac{x^2 - 4x + 4}{x - 3}\, dx$

   (b) $\displaystyle\int \tan 5x\, dx$

   (c) $\displaystyle\int \dfrac{2 + e^x}{e^{2x}}\, dx$

   (d) $\displaystyle\int \dfrac{2}{\sqrt{1 - 4x^2}}\, dx$

   (e) $\displaystyle\int_{-2}^2 \dfrac{1}{4 + x^2}\, dx$

3. (5 pts) Use substitution to find

$$\int_0^{\ln 2} \dfrac{e^{2x}}{2 + e^x}\, dx.$$

4. (8 pts) Let $f(x) = \sqrt{8 - x^3}$, $x \le 2$. Then f is decreasing on $(-\infty, 2]$, so it has an inverse.

   (a) Find $f^{-1}(x)$.

   (b) Find the range of f.

   (c) Find the domain and range of f^{-1}.

   (d) Find the derivative of f^{-1}.

5. (2 pts) Explain why the function $f(x) = x^3 - x$, $0 \le x \le 1$, does not have an inverse.

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6. (10 pts) When one invests money (the principal) at a fixed rate of interest, the interest earned is proportional to the size of the principal. If the interest is added to the principal, then the principal grows at an exponential rate. Suppose that one invests $1000 at a fixed rate of interest such that the principal doubles in 12 years.

    (a) Express the principal P as a function of time t.

    (b) What is the principal after 6 years?

    (c) When will the principal equal $5000?

    (d) What is the rate of change of the principal when $t = 12$?